Leslie Matrix Model and Female Population in Nigeria

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ABSTRACT

In this paper, we verify the accuracy of the Leslie matrix model, which is a bounded by discrete age-structured method row wise and column wise that uses fertility coefficient and survival rates, as a factor for projecting female population. Based on available data for the year 2006, Leslie matrix is built to projects female population in Nigeria for every five years from the years 2006 to 2021. Graphical descriptive statistics were run using Microsoft Excel and SPSS. The results revealed that time specific growth rates (\(\lambda_1, \lambda_2, \lambda_3\)) for 2011, 2016 and 2021 are 14.72\%, 14.50\%, 13.60\% increase, respectively. To test the accuracy of this method, we compared the projected population data for the year 2011 with the estimated population data by National Population Commission for female population in Nigeria.

Keywords: Leslie matrices, fertility coefficient, population.

1. INTRODUCTION

The need for population projection is essential to national planning and development. Government, private sectors, even to researchers, projection of human population is of prime importance. Population projection is a tool for computations of future projection size and characteristics of population based on certain assumptions about future trends in fertility, mortality and migration. Importance of projection is key for housing, employment, estimating the future production of goods and services, future school enrolments, enable planning of investments in school buildings, teacher training and production of educational material, food, power, water, transport, incidence and prevalence of various diseases and planning for the number of hospitals, hospital beds and specialized facilities as well as training programs for medical specialists. In view of the importance and significance of census data, this research work was motivated not only by the failure of Nigeria government in giving attention to decennial conduct of census but also in her failure to conduct credible population census exercise. These are attributed to tribal and political interest of individual in power (government).

Census taking began at least 5800 years ago in Egypt, Babylonia, China, Palestine, and Rome (Halacy, 1980). However, because of the exorbitant cost of conducting census by Governments, the place of population projection came in into play, where Mathematicians, statisticians, demographers have published several works on population projection techniques. In addition, certain complex characteristics like age, sex, are considered in the demographic studies.
It is of strong observation that social researchers of kinds have special interest in the age and sex structure or components of a population, which is attributed to social relationships within a community which are considerably affected by the relative numbers of males and females and the relative numbers at each age (Frank Hobbs, 2004). Moreso, South and Trent (1988) wrote on cross-national analysis of the effect of sex composition on women’s roles, the population growth approximatively correlates to the fertility rates of females in the population. So, female population and the growth within this subgroup, can be used to project population growth. The rest of this research work is structured as follows: Sections 2, 3, and 4 are review on methods adopted in the building of Leslie matrix model, description of data and results. Discussion of results follows while conclusion came last on section 5.

2. METHODS

2.1 Leslie Matrix

When it comes to the study of the growth of a given population it is of great importance having in account the survival and reproduction rates of the individuals of that population. However, these characteristics differ according to the age of the individuals, their body size or any other individual characteristics influencing survival and fertility.

The Leslie matrix model was invented by P. H. Leslie and describes the growth of the female portion of a human or animal population. In this model, the population is divided into groups based on age classes of equal duration. It is a discrete and age structured model.

The purpose is to project the population from time \( t \) to time \( t+1 \), in years or some other time unit, assuming that the unit of time is the same as the age class width (projection interval). A model with projection interval of one year will differ from one that projects from month to month or decade to decade.

Supposing that the individuals of the population are classified into \( k \) age classes, the population projection matrix, often referred as a Leslie matrix, is:

\[
L = \begin{bmatrix}
R_1 & R_2 & R_3 & \ldots & R_{k-1} & R_k \\
S_1 & 0 & 0 & \ldots & 0 & 0 \\
0 & S_2 & 0 & \ldots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & S_{k-1} & S_k
\end{bmatrix}
\]

Where

- \( R \) is the reproduction rate
- \( S \) is the survival rate, of age class \( i \), for \( i=1,\ldots, k \).

In some models the last age class is assumed to be removed from the population after a time unit, so the entry, \( S_k \), is 0 in the matrix \( L \).
If the population at time \( t \), distributed into the \( k \) classes is:

\[
X^{(t)} = \begin{bmatrix}
x_1^{(t)} \\
x_2^{(t)} \\
\vdots \\
x_k^{(t)}
\end{bmatrix}
\]

where \( x_i^{(t)} \) is the population, at time \( t \), in age class \( i=1,..., k \), then the population at time \( t+1 \), in age class \( i=1,..., k \) will be:

\[
x_i^{(t+1)} = R_1 x_1^{(t)} + R_2 x_2^{(t)} + \cdots + R_k x_k^{(t)},
\]

that is, the population in age class one must have originated from reproduction, and not be survivors of any other age class, between times \( t \) and \( t+1 \).

\[
x_{i+1}^{(t+1)} = S_i x_i^{(t)}, \text{ for } i=1,..., k-1,
\]

that is the population in age class \( i+1 \) will be the survivors of class \( i \), between times \( t \) and \( t+1 \).

Using matrix notation the previous equations can be written as \( x^{(t+1)} = LX(t) \).

### 2.1.1 Survival Probabilities

The survival probability depends on the age of the individual from age \( x \) to \( x + 1 \), and is approximated by

\[
P_i = \frac{l(i+1)}{l(i)}
\]

(1)

where the age is assumed to be known (Kot, 2001), if the age is not known (Caswell, 2001), then \( l(x) \) can be approximated by taking its average within each age class over the interval \( i-1 \leq x \leq i \).

Thus,

\[
P_i \approx \frac{l(i) + l(i+1)}{l(i-1) + l(i)}.
\]

(2)

### 2.1.2 Fertilities

Fertility depends on the distribution of births and deaths in the age class, given by,

\[
F = P_{m, i}.
\]

(3)

\( F \) is the number of offspring born in the following year, multiplied by the survival probability.
3. RESULTS

3.1 Data Description:

Data used herein are on the Federal Republic of Nigeria which has an area of 923,769 square kilometers (made up of 909,890 square kilometers of land area and 13,879 square kilometers of water area), which is located between 3° and 14° East Longitude and 4° and 14° North Latitude. The longest distance from East to West is about 767 kilometers, and from North to South 1,605 kilometers. In addition, these data are secondary, which are obtained from Nigeria Demographic and Health Survey 2008, publication The base population data were obtained as a result of population and housing census of 2006, which was carried out from the 21st to 25th March, 2006. The life table from which the survivor ratio was computed was published by World Health Organization.

In the construction of Leslie Matrix Model, from the under listed indices, the components of the Matrix was built, they are:

A life table of female in Nigeria which was obtained from 2006 World Health Organization Report. The importance of this table was to compute survival ratio which is a probability value and placed immediately below the leading diagonal values in the matrix. Age Specific Fertility Rate was obtained from adjusted Age-Sex Specific population distribution of Nigeria for Female (2006). This however was required in order to compute fertility coefficient. Nigeria female population of 2006 was used as the base population upon which population projections were made for 2011, 2016 and 2021. Sex ratio at birth = female birth divided by all birth.

3.2 Results

Projections for the year 2011, 2016 and 2021 were made by using the Leslie matrix constructed from fertility and survival rates in Table 2 versus the data provided for female population as obtained in the 2006 National Population census as shown in table 3. For better comparison of actual (National Population commission) versus projected values, Figure 2 shows how both base population and projected population decrease as age increases. Though, the actual 2011 population data were reported with 65+ as the highest age group where the projected population were reported with 85+ as the highest age group, however it is observed that the data have greater deviation at lower ages (0-4, 5-9, 10-14) and higher ages (60-64, 65+) when presenting the percentage error per age group. In comparison, ages 15-19, 20-24, 25-29, 40-44 and 50-54 have close values for both actual and projected population.
### Table 2: Component of Leslie Matrix Model

<table>
<thead>
<tr>
<th>AGE</th>
<th>FERTILITY COEFFICIENT</th>
<th>SURVIVAL RATIO</th>
</tr>
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<tr>
<td>0-4</td>
<td></td>
<td>0.943976959</td>
</tr>
<tr>
<td>5-9</td>
<td>0.265504367</td>
<td>0.977549852</td>
</tr>
<tr>
<td>10-14</td>
<td>0.190899834</td>
<td>0.941143868</td>
</tr>
<tr>
<td>15-19</td>
<td>0.539274406</td>
<td>0.949122341</td>
</tr>
<tr>
<td>20-24</td>
<td>0.970026888</td>
<td>0.959488526</td>
</tr>
<tr>
<td>25-29</td>
<td>0.328814484</td>
<td>0.97788861</td>
</tr>
<tr>
<td>30-34</td>
<td>0.9781476507</td>
<td>0.981923848</td>
</tr>
<tr>
<td>35-39</td>
<td>0.528814484</td>
<td>0.942448617</td>
</tr>
<tr>
<td>40-44</td>
<td>0.935198124</td>
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</tr>
<tr>
<td>45-49</td>
<td>0.885948215</td>
<td>0.82527354</td>
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<tr>
<td>50-54</td>
<td>0.729144953</td>
<td>0.591047733</td>
</tr>
<tr>
<td>55-59</td>
<td>0.417896608</td>
<td>0.277085219</td>
</tr>
</tbody>
</table>
Figure 1: Leslie Matrix Population Projection Versus NPC Population Projection

Figure 2. Projection error between actual vs. projected population values.
Figure 2: 2006, 2011, 2016 PROJECTED POPULATION COMPARISON

Figure 3: AGE SPECIFIC FERTILITY RATE DISTRIBUTION
4. DISCUSSION

Based on our results, Leslie matrix projections are not reliable for the age groups 0-10 and age groups over 65+ years old, but seemed reliable for the rest of the age groups. This claim is supported by our percent error calculations, which show maximum error in projection to be 27%, whereas at extremes it exceeds well above 50%. The projected female population trend above shows increasing population growth rate as years come by. More so, close examination of population age groups reveals steady increase. The total population size for Nigeria female in 2006 was 69,086,302. The total projected population size for Nigeria female in 2011 as projected was 79,254,392. The time specific growth rate, $\lambda$, is computed as the total population in 2011 divided by the total population in 2006. However, $\lambda = 79,254,392/69,086,302 = 1.1472$. Similarly, total projected population size for Nigeria female in 2016 was 90,749,278. More so, the time specific growth rate, $\lambda$, is computed as the total population in 2016 divided by the total population in 2011 $(90,749,278/79,254,392 = 1.1450)$ which revealed that there is an increase in the population by $(1.1450 \times 100\% = 14.50\%$ increase. Where the time specific growth rate $\lambda$ for 2021 is computed as $(103,117,411 / 90,749,278 = 1.136 x 100\% = 13.60\%$ increase.

<table>
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<tr>
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<td>776513</td>
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<td>97360</td>
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<tr>
<td>TOTAL</td>
<td>69,086,302</td>
<td>79,254,392</td>
<td>81,397,007</td>
<td>90,749,278</td>
<td>103,117,411</td>
</tr>
</tbody>
</table>
5. CONCLUSION

This work has proposed Leslie matrix model for the population projection of Nigeria female. It is observed that little or no application of this model is published to the Nigerian context, so making it unique. Data used to build up this matrix model were obtained from National Population Commission and World Health Organization. Graphical descriptive statistics were run using Microsoft Excel and Statistical Package for Social Science (SPSS) 21. However, the comparison analysis shows that Leslie Matrix projected population are reliable for the rest of the age groups except ages 0-10 and 65+. The time specific growth rate, \( \lambda \), for 2011 showed 14.72% increase and 14.50% increase for 2016, while 2021 would experience 13.60% increase as time specific growth rate.
REFERENCES